WAVE POWER CALCULATIONS

The purpose of this site is to provide a tool for approximate calculations of the power produced by an oscillating buoy. The system, consists of a cylindrical buoy of mass m kg and of diameter D m, oscillating in waves having an amplitude of A m and a period of T seconds. The height of buoy is considered infinite, i.e. the buoyancy is proportional to the depth, no matter how deep under the water surface the buoy goes. This is to ease calculations and tho require one lees input variable from the user.

On the top, the buoy is connected to a generator in such a way that the generator axis moves in both up and down cycles of the buoy. The generator requires a certain torque in order to move and produce electricity. This torque can be translated to a force - according to power generation setup- and this force (G Newton) must be provided by the buoy. If the buoy can not provide this force then neither the buoy nor the generator move. It is further assumed that this force remains steady for as long as the generator axis moves to produce electricity.

In addition, it is possible that a spring device must be used in order to stabilize the motion of the buoy. This spring, having a stiffness of k N/m, is depicted as fixed to the sea bottom. However for the calculations, the spring can be fixed to the top without any influence in the operation. The same holds for the generator, of course which can be placed in the bottom of the sea.

The setup is presented in the following picture.

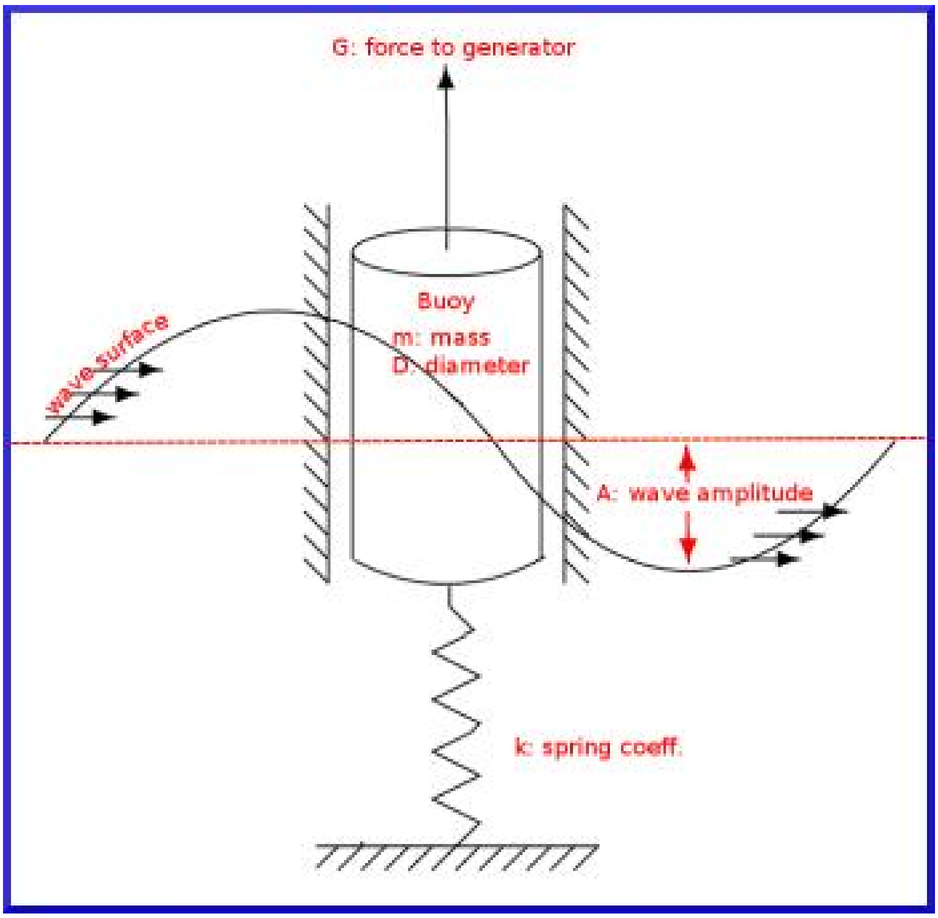

Fig. 1 General setup

MATHEMATICAL ANALYSIS

The initial conditions of the system are depicted in the following figure:

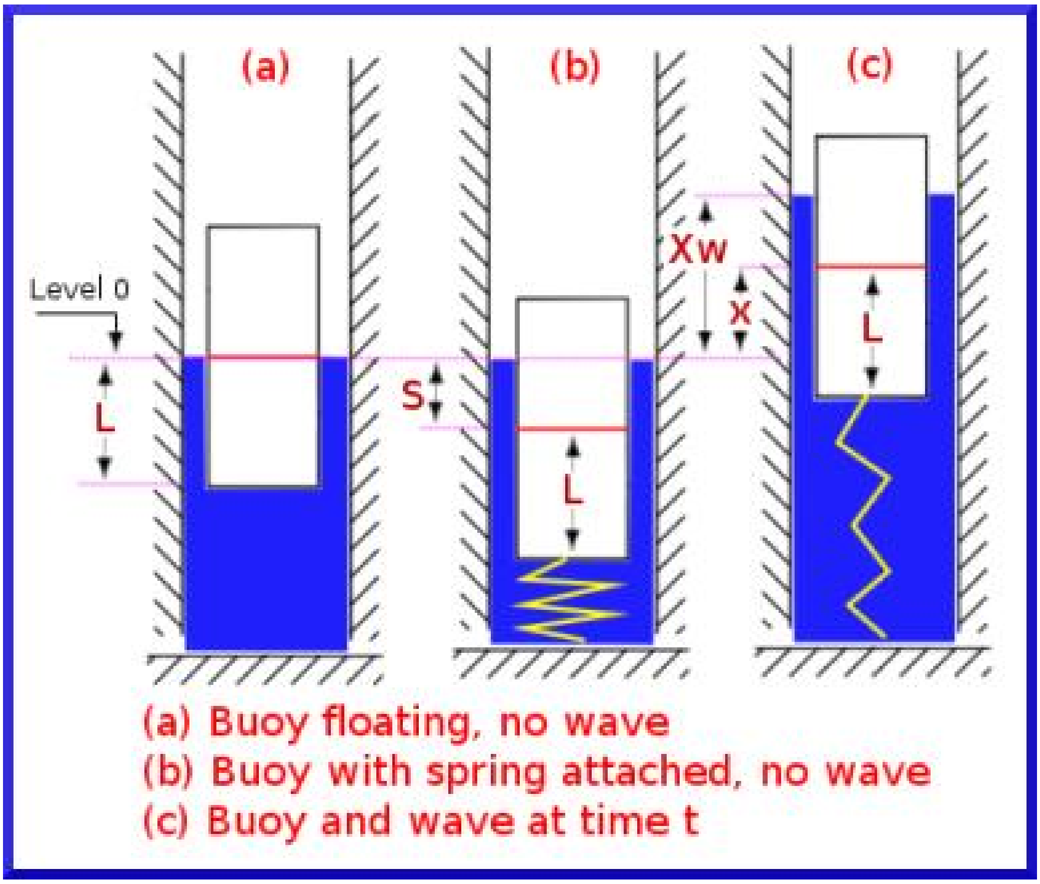

Fig. 2. Initial conditions

In (a) the buoy if floating freely. L is the length of the buoy under water in this position. In (b) a spring is attached to the buoy and is causing the buoy to change position by a distance S (he string is affected by this action, so its length is modified) This is the initial condition. After this, the waves start and the buoy is set in motion.

Considering the positive axis upwards for all vectors like position, speed, force etc, at any moment the force acting on the buoy are:

- The weight of the buoy, W, always steady, always negative.
- The buoyancy, B, proportional to the difference of the water level and the buoy position always positive or zero.
- The generator force, G, always steady and opposite to the velocity of the buoy
- The spring force, Fs, proportional to the distance of the buoy from the spring's original length and opposite to this distance. Note that the spring 's initial length is not the one in the Fig 2.(b) above as in this case the spring is stretched to counteract the extra buoyancy.

Using the symbols in Fig. 2, these forces are:

$W = m * g$ (1)

$$B = \frac{\pi D^2 dg}{4}[Xw - (x - L)]$$

The equilibrium at (a) in Fig. 2 gives $\frac{\pi D^2 dg}{4} L = W = m * g \quad \Rightarrow \quad L = \frac{4}{\pi D^2 dg}$ and the fact that for sinusoidal waves: $Xw = A \sin(\omega t)$ it follows that

$$B = \frac{\pi D^2 dg}{4}\left[A \sin(\omega t) - x + \frac{4}{\pi D^2 dg}\right] \quad (2) \qquad \text{where} \quad \omega = \frac{2\pi}{T} \quad (2a)$$

The generator force is -G when the buoy moves upwards,(positive speed) G when the the buoy moves downwards (negative speed) ans zero when the buoy is stationary.

The spring force, according to Hook's law, is proportional to deflection from its original length .In order to find the position of its length, a correction factor must be added to the distance S of Fig.2. If we call this correction Xc, then $Xo * k = \frac{\pi D^2 dgS}{4}$ because in this position the sprint has to

counteract the combined force of buoyancy minus weight. This leads to $Xo = \frac{\pi D^2 dgS}{4k}$ (2b) and so

the spring force is: $Fs = \left(x - \left(S + \frac{\pi D^2 dgS}{4k}\right)\right)$ (3)

What we are seeking here is a function describing the position of the buoy in time, x(t). The first derivative of x(t) , dx(t)/dt is the velocity (v) and the second derivative $d^2x(t)/dt^2$ is the acceleration (a) of the buoy.

The force acting on the buoy is F = m*a = W + B + G + Fs
Combining the effects of weight and buoyancy from (1) and (2) we get:

$$W + G = \frac{\pi D^2 dg}{4}[A \sin(\omega t) - x]$$

and in order to avoid negative buoyancy in the calculations in case the buoy is ejected out of the water:

$$W + G = max\left[\frac{\pi D^2 dg}{4}[\sin(\omega t) - x], \quad -mg\right]$$

This fact must be also anticipated in the spring correction factor which will be calculated as :

$$Xo = \frac{min\left(\frac{\pi D^2 dgS}{4}, mg\right)}{k} \quad (3a)$$

The final equation is derived by applying Newton's law of motion for the total forces acting on the buoy:

$$m\frac{d^2x(t)}{dt^2}=max\left[\frac{\pi D^2 dg}{4}[\sin(\omega t)-x],-mg\right]-k(x-(S+Xo))-G\begin{cases}\frac{dx(t)/dt}{|dx(t)/dt|} \text{ when } d(x)/dt\neq 0\\ 0 \text{ when } d(x)/dt=0\end{cases}$$

where: x(t) is the buoy position as a function of time t, m is the mass of the buoy, D is the diameter of the buoy, d is the density of sea water (taken as 1025 kg/m3), g is the acceleration of gravity (9.81 m/s^2) , A is the amplitude of the waves, T is the period , k is the spring stiffness and G is the generator force. ω is the radial speed provided by (2a) above.

This second order differential equation can be reduced to a system of two first order equations by putting dx(t)/dt= u. Dividing by m both terms of the equation and rearranging, this system is:

$$\frac{dx(t)}{dt}=u \quad (4) \text{ and}$$

$$\frac{d(u)}{dt}=max\left[\frac{\pi D^2 dg}{4m}[\sin(\omega t)-x],-g\right]-\frac{k(x-(S+Xo))}{m}-\frac{G}{m}\begin{cases}\frac{u}{|u|} \text{ when } u\neq 0\\ 0 \text{ when } u=0\end{cases} \quad (5)$$

The system of (4) and (5) can be solved numerically by using a fourth order Runge – Kutta algorithm. Xo is calculated from (3b).

Added mass, water friction and generator inertia were not considered at this stage but can be added easily to (5) above if the coefficients are known.

The suitability of the Runge – Kutta algorithm was tested by solving numerically the equation of motion without generator and initial deflection by the spring:

$$m\frac{d^2x(t)}{dt^2}=\frac{\pi D^2 dg}{4}[\sin(\omega t)-x]-kx$$

This he equation can be easily solved theoretically and the result is:

$$x(t)=\frac{A\sqrt{\Omega}}{\Omega-\omega^2}\left(\sqrt{\Omega}\sin(\omega t)-\omega\sin(\sqrt{\Omega}t)\right) \quad (6)$$

where: $\Omega=\frac{\pi D^2 dg}{4m}+\frac{k}{m}$ and $\omega=\frac{2\pi}{T}$ (7)

The algorithm gives very good fit for most reasonable input ranges, if the step is lower than 0.01 sec. A small but noticeable error was observed at the point where the speed is maximum , around x(t)=0.

USE OF THE PROGRAM

After the explanations above, the program should be easy to operate. The user should not enter values for the time period and the step that result in a high number of steps (>100000) in order to avoid exhausting the system memory for the huge arrays needed for graph creation. However, when the box [Higher precision] is ticked, the program uses internally up to 1.000.000 steps in order to increase accuracy, but this results in increased wait time. The size of the graph can vary from 300X600 to 5000X20000 pixels.

The option [optimize mass], when ticked, makes the program ignore the mass entered by the user and calculates the mass at which resonance occurs. This happens when $\sqrt{\Omega}=\omega$ as defined in (7) and this means :

$$m=\frac{T^2}{16\pi^2}\left(\pi D^2 dg+4k\right)$$

The user can save the produced graph as a normal png image.

As an alternative to sinusoidal waves the user can choose square or triangular waves with the same amplitude and period.

The energy shown in the graph as negative, is the energy produced during the "downward" phase of the buoy.

When the box [Produce calculation file] is ticked the program creates a file with the data of calculation steps, which the user can view/download by clicking on a button on the graph.

BIBLIOGRAPHY

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